

2002 FP2 Adapted

1. Find the set of values for which

$$|x - 1| > 6x - 1. \quad (5)$$

2. (a) Find the general solution of the differential equation $t \frac{dv}{dt} - v = t, \quad t > 0$

and hence show that the solution can be written in the form $v = t(\ln t + c)$, where c is an arbitrary const. (6)

(b) This differential equation is used to model the motion of a particle which has speed $v \text{ m s}^{-1}$ at time $t \text{ s}$. When $t = 2$ the speed of the particle is 3 m s^{-1} . Find, to 3 sf, the speed of the particle when $t = 4$. (4)

3. (a) Show that $y = \frac{1}{2}x^2e^x$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = e^x. \quad (4)$$

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = e^x$.

given that at $x = 0, y = 1$ and $\frac{dy}{dx} = 2$. (9)

4. The curve C has polar equation $r = 3a \cos \theta, -\frac{\pi}{2} \leq \frac{\pi}{2}$.

The curve D has polar equation $r = a(1 + \cos \theta), -\pi \leq \theta < \pi$. Given that a is a positive constant,

(a) sketch, on the same diagram, the graphs of C and D , indicating where each curve cuts the initial line. (4)

The graphs of C intersect at the pole O and at the points P and Q .

(b) Find the polar coordinates of P and Q . (3)

(c) Use integration to find the exact area enclosed by the curve D and the lines $\theta = 0$ and $\theta = \frac{\pi}{3}$ (7)

The region R contains all points which lie outside D and inside C .

Given that the value of the smaller area enclosed by the curve C and the line $\theta = \frac{\pi}{3}$ is

$$\frac{3a^2}{16}(2\pi - 3\sqrt{3}),$$

(d) show that the area of R is πa^2 . (4)

5. Using algebra, find the set of values of x for which $2x - 5 > \frac{3}{x}$.
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6. (a) Find the general solution of the differential equation

$$\cos x \frac{dy}{dx} + (\sin x)y = \cos^3 x. \quad (6)$$

(b) Show that, for $0 \leq x \leq 2\pi$, there are two points on the x -axis through which all the solution curves for this differential equation pass. (2)

(c) Sketch the graph, for $0 \leq x \leq 2\pi$, of the particular solution for which $y = 0$ at $x = 0$. (3)

7. (a) Find the general solution of the differential equation

$$2 \frac{d^2 y}{dt^2} + 7 \frac{dy}{dt} + 3y = 3t^2 + 11t. \quad (8)$$

(b) Find the particular solution of this differential equation for which $y = 1$ and $\frac{dy}{dt} = 1$ when $t = 0$. (5)

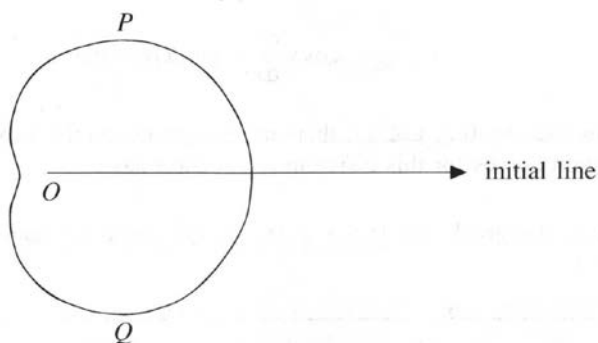
(c) For this particular solution, calculate the value of y when $t = 1$. (1)

8.

Figure 1

The curve C shown in Fig. 1 has polar equation

$$r = a(3 + \sqrt{5} \cos \theta), \quad -\pi \leq \theta < \pi.$$



(a) Find the polar coordinates of the points P and Q where the tangents to C are parallel to the initial line. (6)

The curve C represents the perimeter of the surface of a swimming pool. The direct distance from P to Q is 20 m.

(b) Calculate the value of a . (3)

(c) Find the area of the surface of the pool. (6)

9. (a) The point P represents a complex number z in an Argand diagram. Given that

$$|z - 2i| = 2|z + i|,$$

(i) find a cartesian equation for the locus of P , simplifying your answer. (2)

(ii) sketch the locus of P . (3)

(b) A transformation T from the z -plane to the w -plane is a translation $-7 + 11i$ followed by an enlargement with centre the origin and scale factor 3.

Write down the transformation T in the form

$$w = az + b, \quad a, b \in \mathbb{C}. \quad (2)$$

10.

$$y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + y = 0.$$

(a) Find an expression for $\frac{d^3 y}{dx^3}$. (5)

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$,

(b) find the series solution for y , in ascending powers of x , up to an including the term in x^3 . (5)

(c) Comment on whether it would be sensible to use your series solution to give estimates for y at $x = 0.2$ and at $x = 50$. (2)

Total 112 marks